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Book Review

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Optimal Control

Richard Vinter, Birkhäuser, 2000, $79.95, ISBN 0-8176-4075-4

Optimal control theory is a prime example of interdependence of mathematics and engineering. The early motivation for development of this field in the late 1950s and early 1960s lay in aerospace engineering applications. In fact, formulation of the Maximum Principle by Pontryagin and his associates at Steklov Institute in Moscow was in large part due to dedication of these "pure" mathematicians to solve applied engineering problems.

Since then, the field has matured because of continued efforts of mathematicians and engineers alike. The two different methodologies used in studying optimal control problems, namely the Maximum Principle and dynamic programming, both have advanced from the analytical tools provided by "nonsmooth analysis" which was pioneered by F. H. Clarke in the 1970s.

It is known that for many control problems the solutions to the Hamilton–Jacobi equation in dynamic programming have discontinuous derivatives. It is also known that the original form of the Pontraygin Maximum Principle can not handle the case of nonsmooth data. In aerospace applications, nonsmoothness appears in examples such as launch problems where discontinuity of state variables (mass) or dynamics are expected. In optimization problems any cost function that depends on something as simple and natural as the distance function requires nonsmooth formulation. As Clarke himself has noted, nonsmooth analysis is motivated not just by natural occurrence of nonsmooth (nondifferentiable) phenomena, but also by its usefulness in analyzing smooth problems in a more general setting. Nonsmooth analysis may in fact be what has bridged the classical calculus of variations which depends so much on smoothness properties of variables and the modern optimal control theory.

To account for all the developments in the field and bring together the different facets of the results in one volume is no small feat, but who is better suited to achieve this task than a leading mathematician in the field such as Richard Vinter? He has put together the important advances of the last two decades interspersed with enough background information and technical detail to make the book accessible to a mathematically mature audience who is not necessarily familiar with the field of optimal control theory. It is helpful for the reader to be familiar with the basic concepts of real and functional analysis. In the preface, Vinter explains that he tries to provide "quick answers to the questions: what are the main results, what were the deficiencies of the 'classical' theory and to what extent they have been overcome?" The first chapter of the book addresses these questions in an expository style and provides an informative background. The epilogue at the end of this chapter also provides a good overview of optimal control theory in general and contains many references for the interested reader. In fact, this chapter and its epilogue should be used as a road map and general guideline to the mathematically intricate world of *Optimal Control*.

The book begins its more detailed discussion of the subjects presented in Chapter 1, by discussing measurable multifunctions and differential inclusions in Chapter 2. While this chapter is in no way exhaustive on the vast subject of differential inclusions, its discussion may be more general and mathematical than the existing discussion of the subject in the AIAA optimal control theory community. Perhaps to motivate the reader to forge through the mathematical maze of this chapter, it should be noted that differential inclusions are the natural setting for commonly occuring problems with constraints on the velocity variable. Even without reading all the mathematical details in the proofs, a cursory reading of the general results provides the reader with the basic idea behind this formulation. The third chapter discusses variational principles which are of great importance to nonsmooth analysis. The Exact Penalization Theorem is discussed, including its role in the derivation of necessary conditions of constrained optimization problems. For the reader with interest in optimization applications, a min-max theorem due to Von Neumann concludes the section.

Chapters 4 and 5 provide the nitty-gritty details of nonsmooth analysis. For the reader with interest in knowing the Nonsmooth Maximum Principle results, learning the basic tools and ideas of these chapters such as normal cones, subdifferentials, the distance functions, and so on, is essential.

Formulation of the Maximum Principle is what propelled optimal control theory as a distinct field, and therefore, the extensive treatment it receives in Vinter's book is fully expected. Even though the results are generally known as the Pontraygin Maximum Principle, there is some discussion on who should be credited as the primary author on the subject. The results were published in Russian in 1961 in a book authored by Pontraygin, Boltyanskii, Gamkrelidze, and Mischenco and the first proof is attributed to Boltyanskii. Chapter 6 in this book covers the history and discusses the basic formulation of the "smooth" Maximum Principle and its proof. As explained in the notes at the end of this chapter, essentially three different versions of the Maximum Principle are proved, namely "for problems with smooth data and free right endpoints, for problems with smooth data and general endpoint constraints, and, finally for nonsmooth problems with general constraints." This discussion is followed in the next three chapters where necessary conditions for optimal control problems formulated as differential inclusions are developed as well as for free end-time and state constrained problems. The most recent results for the nonsmooth Maximum Principle (mostly derived by F. H. Clarke and the author himself) are summarized in these chapters.

The last two chapters cover some interesting and important subjects: Regularity of Minimizers and Dynamic Programming. The first topic connects the discussion on optimal control to the broader aspects in optimization theory. Regularity analysis of minimizers has important applications: As stated in Chapter 11, one implication is to help us "identify classes of problems, for which all minimizers satisfy some known necessary conditions of optimality." This identification justifies the search among the extremals for the minimizers. The author has given an example in Chapter 1, where the unique extremal solution that satisfies the necessary conditions is not a minimizer. To address this issue, in the early 1900s, Tonelli considered establishing existence of minimizers before identifying them using necessary optimality conditions. His existence theorems and their extensions are discussed in great detail in Chapter 11. The other important application of regularity theory is that by identifying the class of functions to which the minimizers belong, one can choose numerical methods and discretizations for solving these problems that preserve the regularity properties of these solutions. This point is of great significance to the field of numerical optimal control theory.

In the last chapter, discussing dynamic programming and generalized solutions to Hamilton–Jacobi equations brings together the two new developments of the modern optimal control theory. As mentioned in the preface the discussion is by no means all-inclusive and is instead concentrated on aspects of dynamic programming related to the topics already discussed in the book. To know more on the viscosity methods, the reader should refer to the vast literature on the subject.

Optimal Control by Richard Vinter deserves the attention of researchers in the field of optimal control theory as well as the novice readers who might foresee the far-reaching applications of optimal control and nonsmooth analysis in their own respective field. The book is mathematically rich and detailed but its first chapter and the notes at the end of each chapter provide a comprehensive and detailed overview of the subject matter. With some patience in acquainting oneself with the basic mathematical tools, reading this book promises to be a valuable learning and reading experience.

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